order $p^n$. Let $h_{i_1}\ h_{i_2}$ ..... represent the subgroups of $H$, of order $p$, and $J_{ik}=\begin{pmatrix} P_1 & P_2 & \ldots\ldots P_n \\ h_{i_1} & h_{i_2} & \ldots\ldots h_{i_n} \end{pmatrix}$ the isomorphism of $H$ gotten by replacing $P_j$ by any operation (order $p$) in $h_{ij}$ ($j=1\ 2\ \ldots\ldots m$), say the new generators from the $k$th set of all of the possible sets which might be chosen from $h_{i_1}$, $h_{i_2}$ ..... $h_{i_n}$. The number of values of $k$ is obviously equal to $\Phi(p)^n=(p-1)^n$. To determine the number of choices *of this set* of subgroups (number of values of $i$) suppose that $\alpha$ of a set of $n$ generators have been selected. The remaining $n-\alpha$ operations must be selected outside the subgroup $H_{p^\alpha}$ generated by the first $\alpha$, and thus there remain

$$\frac{p^n-1}{p-1}-\frac{p^\alpha-1}{p-1}=\frac{p^\alpha(p^{n-\alpha}-1)}{p-1}$$

subgroups $h_{ij}$ from which to select the remaining $n-\alpha$. Thus the product of the number of values of $k$ and the number of values of $i$ is

$$h=(p-1)^n\prod_{\alpha=0}^{n-1}\frac{p^\alpha(p^{n-\alpha}-1)}{p-1}=(p^n-1)(p^n-p)(p^n-p^2)\ldots\ldots(p^n-p^{n-1})$$

which is the number of choices of new generators of $H$, or the order of its automorph.

## MECHANICS.

186. Proposed by R. D. CARMICHAEL, Hartselle, Alabama.

A point $P$ keeps at uniform distance from and moves with uniform angular velocity around a point $Q$ which is in harmonic motion, making one revolution while $Q$ swings to and fro. If $P$ is in the line of the path of $Q$ and on the same side of the center of that path with $Q$ when $Q$ is at the extremity of the path, what is the locus of $P$?

Solution by the PROPOSER.

Take the origin at the center of the path of $Q$, and let $a$=half the length of that path. Let $PQ=b$, and let $\theta$=the angle of $PQ$ with the path of $Q$ at any time. Then, it is easily shown that $x=(a+b)\cos\theta$, $y=b\sin\theta$, the equations of an ellipse whose axes are $a+b$ and $b$.

Also solved by G. W. Greenwood, and G. B. M. Zerr.